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What if the mass difference ΔM_s is around 18 inverse picoseconds?

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Abstract. Present experiments in pursuit of the mass difference in the B_s^0-$\overline{B_s^0}$ system have put a lower bound on this quantity of $\Delta M_s > 14.9$ ps^{-1} (at 95% C.L.). The same experiments also yield a local minimum in the log-likelihood function around $\Delta M_s = 17.7$ ps^{-1}, which is 2.5σ away from being zero. Motivated by these observations, we investigate the consequences of a possible measurement of $\Delta M_s = 17.7 \pm 1.4$ ps^{-1}, in the context of both the standard model and supersymmetric models with minimal flavor violation. We perform a fit of the quark mixing parameters in these theories and estimate the expected ranges of the CP asymmetries in B decays, characterized by α, β and γ, the interior angles of the CKM-unitarity triangle. Based on this study, we argue that, if indeed ΔM_s turns out to be in its currently-favored range, this would disfavor a large class of supersymmetric models. Indeed, of all the models examined here, the best fit to the data occurs for the standard model.

1 Introduction

One of the principal aims of flavor physics is to measure the parameters of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1], which encodes the manner in which quark mixing takes place within the Standard Model (SM). There are many measurements which contribute to this goal. For example, the matrix element V_{ud} can be probed through the study of neutron β decay, while the B_d^0–$\overline{B_d^0}$ mass difference ΔM_d can be used to determine the matrix element V_{td}. Our present knowledge of the CKM matrix is usually displayed in terms of the allowed region of the so-called unitarity triangle [2]. Ongoing experiments studying B-hadron physics will be able to test the CKM matrix by measuring the sides and the (CP-violating) angles of the unitarity triangle. If physics beyond the SM is present, inconsistencies in the various unitarity tests will appear. If this occurs, then it will be necessary to perform an overall fit of the CKM matrix elements in various competing theories in order to establish the right framework for flavor physics.

One appealing candidate theory which may induce such "unitarity inconsistencies" is supersymmetry (SUSY). In its minimal flavor-violating form, the couplings of SUSY particles to ordinary matter are proportional to CKM matrix elements. Thus, the weak phases of supersymmetric contributions to loop-induced transitions are the same as in the SM. These loop-level processes include B_d^0–$\overline{B_d^0}$ and B_s^0–$\overline{B_s^0}$ mixing, as well as the flavor-changing neutral-current decays $b \to s\gamma$ and $b \to s\ell^+\ell^-$. The presence of such additional SUSY contributions has the effect that the extracted values of the matrix elements $|V_{td}|$ and $|V_{ts}|$ will be modified from their SM values. Conversely, precise measurements of the CKM matrix elements may put severe bounds on new physics, including SUSY. In [3], we demonstrated this quantitatively: we worked out the profile of the CKM unitarity triangle in the SM and in several variants of minimal flavor-violating supersymmetric models. We also examined the correlations among the CP-violating phases α, β and γ in these models. Although, at the present time, all models give reasonable fits to the data, in the future, with more precise data, one will be able to distinguish among the various candidate models.

If one compares the allowed region of the unitarity triangle of today with that of the early 1990's [4] it is clear that the current region is considerably smaller. Although the errors on virtually all measurements have decreased since the early 1990's, the single most important improvement has been the measurement of ΔM_s in B_s^0–$\overline{B_s^0}$ mixing. As the lower limit on ΔM_s has increased over the years, more and more of the earlier-allowed region has been cut away. Indeed, this lower limit continues to increase: although the lower limit in 1999 was $\Delta M_s > 12.4$ ps^{-1}, it now stands at $\Delta M_s > 14.9$ ps^{-1} [5]. More intriguing, there is now a hint of a possible signal at $\Delta M_s \simeq 17.7$ ps^{-1} [5]. Clearly, the last word on ΔM_s from the combined LEP/SLD/CDF analysis is yet to come, and it is conceivable that the measurement of ΔM_s is just around the corner. In anticipation of this, and to underscore the importance of the ΔM_s measurement for CKM phenomenology, in this paper we present two analyses. First, we up-

date the CKM fits in the SM and in the supersymmetric models mentioned above. Second, we assume a (future) measurement of $\Delta M_s = 17.7 \pm 1.4$ ps^{-1}, and examine the consequences. As we will see, such a measurement would be sufficient to disfavor a large class of minimal flavor-violating supersymmetric models (though it would be completely consistent with the SM).

The paper is organized as follows. In the next section, we present the year-2000 profile of the unitarity triangle, both in the SM and in supersymmetric theories with minimal flavor violation. As we will see, the measurement of β will not distinguish among these various models, though the measurement of α and/or γ will. More to the point, in supersymmetric models one can obtain a very different allowed range for ΔM_s, so that a precision measurement of this quantity will be able to strongly constrain the SUSY parameter space. This is shown quantitatively in Sect. 3. Here we present a future profile of the unitarity triangle, both in the SM and in SUSY, assuming a hypothetical measurement of $\Delta M_s = 17.7 \pm 1.4$ ps^{-1}. Such a measurement would disfavor a certain class of SUSY models. Furthermore, it turns out that, of all the models considered here, the SM yields the best fit to the data. Thus, the hint of a signal at $\Delta M_s = 17.7$ ps^{-1} is not in any way in conflict with the SM. (Of course, there is still a large class of SUSY models which provides a reasonable fit to the data.) We conclude in Sect. 4.

2 Unitarity triangle: Year-2000 profile

It is customary to use an approximate parametrization of the CKM matrix, due to Wolfenstein [6], to quantitatively discuss the allowed region of the unitarity triangle. The Wolfenstein parametrization can be written as

$$V \simeq \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda(1+iA^2\lambda^4\eta) & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} . \quad (1)$$

Thus, λ, A, ρ and η are the four quantities which parametrize the CKM matrix.

With the experimental precision expected in future B (and K) decays, it may become necessary to go beyond leading order in λ in the Wolfenstein parametrization given above. To this end, we follow here the prescription of Buras et al. [7]: defining $\bar\rho \equiv \rho(1-\lambda^2/2)$ and $\bar\eta \equiv \eta(1-\lambda^2/2)$, we have

$$\begin{aligned} &V_{us} = \lambda, \quad V_{cb} = A\lambda^2, \\ &V_{ub} = A\lambda^3(\rho - i\eta), \\ &V_{td} = A\lambda^3(1-\bar\rho - i\bar\eta) \, . \end{aligned} \quad (2)$$

(Note that the matrix elements V_{us}, V_{cb} and V_{ub} remain unchanged, but V_{td} is renormalized in going from leading order to next-to-leading order.) The apex of the unitarity triangle is now defined by the renormalized Wolfenstein parameters $(\bar\rho, \bar\eta)$.

Table 1. Data used in the CKM fits

Parameter	Value
λ	0.2196
$\lvert V_{cb}\rvert$	0.0404 ± 0.0018
$\lvert V_{ub}/V_{cb}\rvert$	0.087 ± 0.018
$\lvert\epsilon\rvert$	$(2.280 \pm 0.013) \times 10^{-3}$
ΔM_d	0.487 ± 0.014 ps^{-1}
ΔM_s	> 14.9 ps^{-1}
$\overline{m_t}(m_t(pole))$	165 ± 5 GeV
$\overline{m_c}(m_c(pole))$	1.25 ± 0.05 GeV
$\hat\eta_B$	0.55
$\hat\eta_{cc}$	1.38 ± 0.53
$\hat\eta_{ct}$	0.47 ± 0.04
$\hat\eta_{tt}$	0.57
$\hat B_K$	0.94 ± 0.15
$f_{B_d}\sqrt{\hat B_{B_d}}$	230 ± 40 MeV
ξ_s	1.16 ± 0.05

2.1 Input data

There are a variety of measurements which constrain $\bar\rho$ and $\bar\eta$, either directly or indirectly. The theoretical and experimental quantities which are used in the CKM fits are listed in Table 1, along with their present values and errors (if applicable). For a detailed description of these quantities, as well as a discussion of our methodology, we refer the reader to [3].

The one measurement which must be described in more detail here is ΔM_s. Since the first studies of B_s^0-$\overline{B_s^0}$ mixing in the SM [8], it was known that the measurement of the mass differences ΔM_s and ΔM_d would provide a powerful constraint on the CKM matrix elements. The ratio of these mass differences can be expressed in the SM as:

$$\frac{\Delta M_s}{\Delta M_d} = \frac{\hat\eta_{B_s} M_{B_s}\left(f_{B_s}^2 \hat B_{B_s}\right)}{\hat\eta_{B_d} M_{B_d}\left(f_{B_d}^2 \hat B_{B_d}\right)} \left|\frac{V_{ts}}{V_{td}}\right|^2 = C\frac{\xi_s^2}{\lambda^2}\frac{1}{(1-\bar\rho)^2+\bar\eta^2}. \quad (3)$$

Since the QCD correction factors satisfy $\hat\eta_{B_s} = \hat\eta_{B_d} = 0.55$ [9], and since $C = M_{B_s}/M_{B_d} = 1.017$ [2], the only real uncertainty in this quantity is the ratio of hadronic matrix elements $\xi_s \equiv (f_{B_s}\sqrt{\hat B_{B_s}})/(f_{B_d}\sqrt{\hat B_{B_d}})$. It is now widely accepted that the ratio ξ_s is probably the most reliable of the lattice-QCD estimates in B physics, $\xi_s = 1.16 \pm 0.05$ [10]. Thus, the accurate knowledge of $\Delta M_s/\Delta M_d$ puts a stringent constraint on the CKM parameters $\bar\rho$ and $\bar\eta$, and hence on the allowed region of the unitarity triangle.

Since ΔM_d has already been measured very accurately (the present world average is $\Delta M_d = 0.487 \pm 0.014$ ps^{-1} [5]), a measurement of ΔM_s is being keenly awaited. The present experimental situation on ΔM_s can be summarized as follows: the combined analysis of the LEP/SLD/CDF measurements undertaken by the B-oscillation working group yields a lower bound $\Delta M_s > 14.9$ ps^{-1} (at 95%

C.L.) [5], using the amplitude analysis method of Moser and Rousarie [11]. However, quite interestingly, the same analysis also yields a *local minimum in the log-likelihood distribution around* $\Delta M_s = 17.7$ ps^{-1}, whose significance becomes more pronounced if the amplitude spectrum is converted to a log-likelihood function referenced to $\Delta M_s = \infty$: $\Delta \log L^{\infty}(\Delta M_s) = (0.5 - \mathcal{A})/\sigma^2_{\mathcal{A}}$ [12]. Here $\mathcal{A}$ is an amplitude modulating the oscillating terms as $(1 \pm \mathcal{A} \cos \Delta M_s t)$, with $\sigma_{\mathcal{A}}$ being its error. This local minimum has the interpretation that at this value of ΔM_s, the amplitude $\mathcal{A}$ is away from being zero (no-mixing case) by 2.5σ. The statistical significance of this result has been studied in a monte-carlo based analysis by Boix and Abbaneo [13]. They estimate the probability that the observed result was produced by a statistical fluctuation anywhere in the scanned values of ΔM_s to be $1 -$ C.L. $\simeq 2.5\%$ [5]. Although this probability is not yet small enough to consider this to be a measurement of ΔM_s, the result is intriguing.

The other quantity which must be mentioned is $\sin 2\beta$. Since a non-zero value of $\sin 2\beta$ would be the first evidence for CP violation outside the kaon system, many experiments are attempting to measure this quantity. In the Wolfenstein parametrization, $-\beta$ is the phase of the CKM matrix element V_{td}. From (1) one can readily find that

$$\sin 2\beta = \frac{2\bar{\eta}(1-\bar{\rho})}{(1-\bar{\rho})^2 + \bar{\eta}^2} \ . \tag{4}$$

Thus, a measurement of $\sin 2\beta$ would put a strong constraint on the parameters $\bar{\rho}$ and $\bar{\eta}$.

In fact, first measurements of $\sin 2\beta$ have already been reported, and the present status is summarized below:

$$\begin{aligned} \sin 2\beta &= 3.2^{+1.8}_{-2.0} \pm 0.5 \text{ (OPAL [14])}, \\ &= 0.79^{+0.41}_{-0.44} \text{ (CDF [15])}, \\ &= 0.84^{+0.83}_{-1.04} \pm 0.16 \text{ (ALEPH [16])}, \\ &= 0.45^{+0.43\ +0.07}_{-0.44\ -0.09} \text{ (BELLE [17])}, \\ &= 0.12 \pm 0.37 \pm 0.09 \text{ (BABAR [18])}, \end{aligned} \tag{5}$$

yielding a world average $\sin 2\beta = 0.48^{+0.22}_{-0.24}$ [19]. This quantity will eventually be very precisely measured at the ongoing B-factory experiments and elsewhere. However, since the error is still quite large, we do not include this measurement in our fits.

2.2 SM fits

In order to find the allowed region in $\bar{\rho}$–$\bar{\eta}$ space, i.e. the allowed shapes of the unitarity triangle, the computer program MINUIT is used to fit the parameters to all the experimental constraints. In the fit, we allow ten parameters to vary: $\bar{\rho}$, $\bar{\eta}$, A, m_t, m_c, η_{cc}, η_{ct}, $f_{B_d}\sqrt{\hat{B}_{B_d}}$, $\hat{B}_K$, and ξ_s. The ΔM_s constraint is included using the amplitude method [11]. The allowed (95% C.L.) $\bar{\rho}$–$\bar{\eta}$ region is shown in Fig. 1. The triangle drawn is to facilitate our discussions, and corresponds to the central values of the fits, $(\alpha, \beta, \gamma) = (95°, 22°, 63°)$.

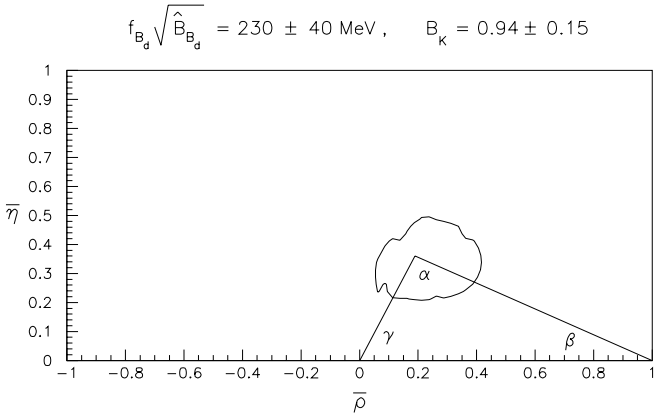

Fig. 1. Allowed region in $\bar{\rho}$–$\bar{\eta}$ space in the SM, from a fit to the ten parameters discussed in the text and given in Table 1. The solid line represents the region with $\chi^2 = \chi^2_{min} + 6$ corresponding to the 95% C.L. region. The triangle shows the best fit

The CP angles α, β and γ can be measured in CP-violating rate asymmetries in B decays. These angles can be expressed in terms of $\bar{\rho}$ and $\bar{\eta}$. Thus, different shapes of the unitarity triangle are equivalent to different values of the CP angles. Referring to Fig. 1, the allowed ranges at 95% C.L. are given by

$$77° \le \alpha \le 127° \ , \quad 14° \le \beta \le 35° \ , \quad 34° \le \gamma \le 81° \ , \tag{6}$$

or, equivalently,

$$\begin{aligned} -0.96 \le \sin 2\alpha \le 0.45 \ , \\ 0.46 \le \sin 2\beta \le 0.94 \ , \\ 0.31 \le \sin^2 \gamma \le 0.98 \ . \end{aligned} \tag{7}$$

Of course, the values of α, β and γ are correlated, i.e. they are not all allowed simultaneously. After all, the sum of these angles must equal 180°. We illustrate these correlations in Figs. 2 and 3. In both of these figures, the SM plot is labelled by $f = 0$. Figure 2 shows the allowed region in $\sin 2\alpha$–$\sin 2\beta$ space allowed by the data, while Fig. 3 shows the allowed (correlated) values of the CP angles α and γ. This correlation is roughly linear, due to the relatively small allowed range of β (6).

Finally, one can also calculate the range of ΔM_s which is presently allowed in the SM. At 95% C.L. we find:

$$14.6 \le \Delta M_s \le 31.2 \ . \tag{8}$$

2.3 SUSY fits

In this subsection we update the profile of the unitarity triangle in supersymmetric (SUSY) theories with minimal flavor violation. In this class of models, the SUSY contributions to ΔM_d, ΔM_s and $|\epsilon|$ can all be described by a single common parameter f (for a more detailed discussion, we refer the reader to [3]):

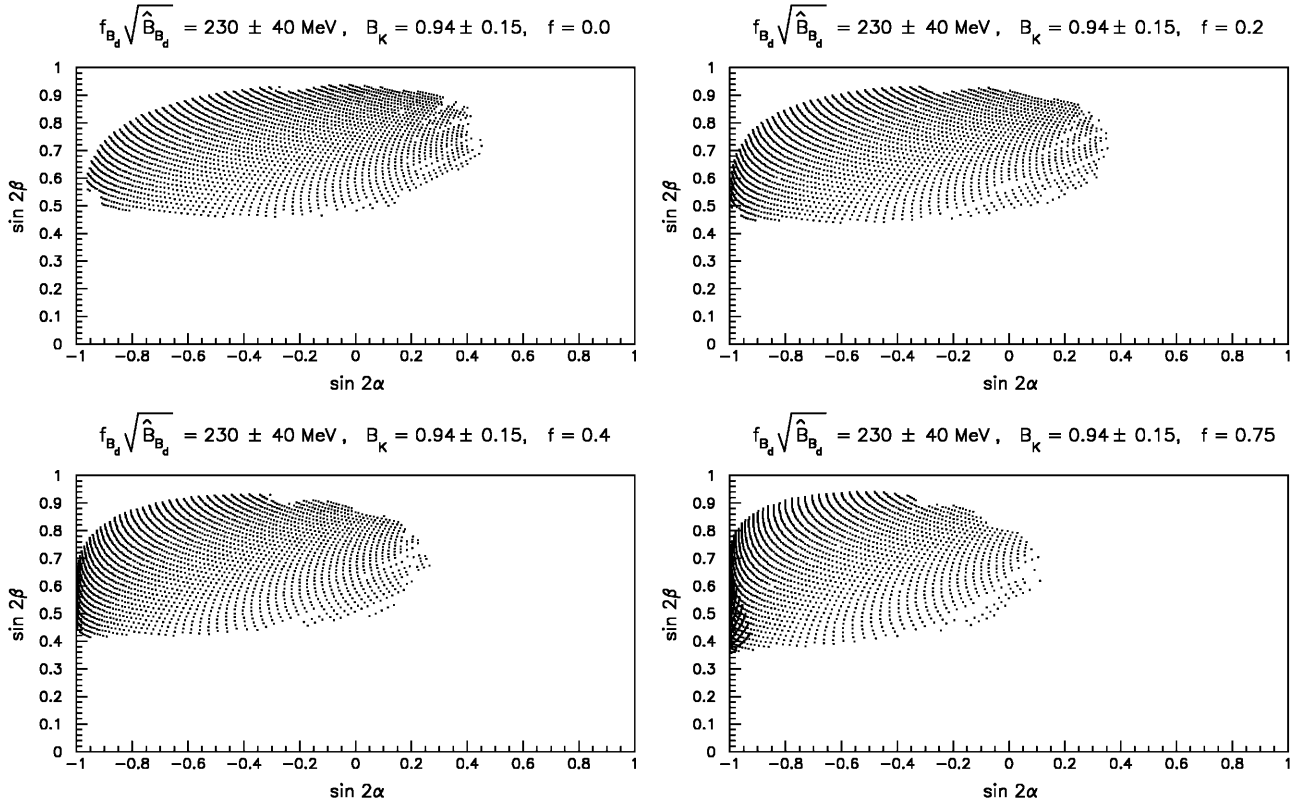

Fig. 2. Allowed 95% C.L. region of the CP-violating quantities $\sin 2\alpha$ and $\sin 2\beta$, from a fit to the data given in Table 1. The upper left plot ($f = 0$) corresponds to the SM, while the other plots ($f = 0.2$, 0.4, 0.75) correspond to various SUSY models

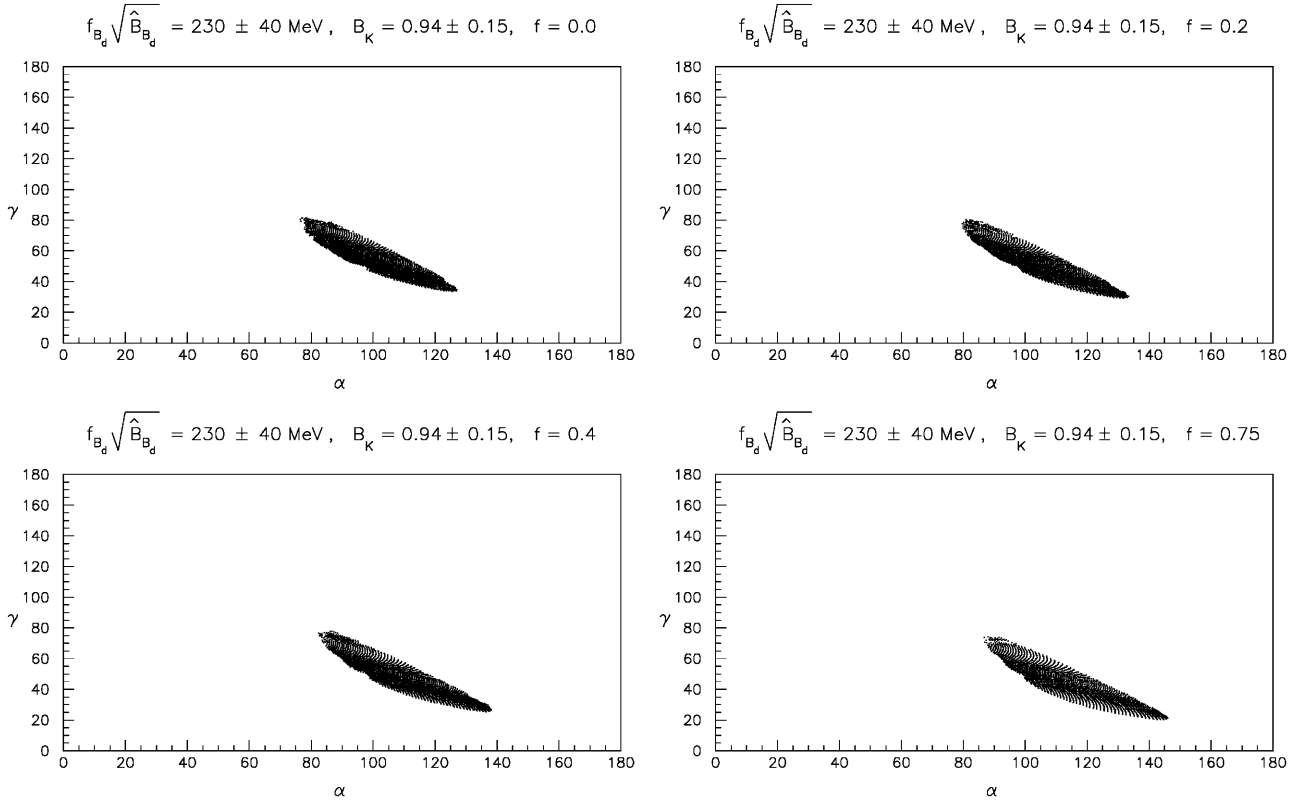

Fig. 3. Allowed 95% C.L. region of the CP-violating quantities α and γ, from a fit to the data given in Table 1. The upper left plot ($f = 0$) corresponds to the SM, while the other plots ($f = 0.2$, 0.4, 0.75) correspond to various SUSY models

$$\begin{aligned}\Delta M_d &= \Delta M_d(SM)[1+f],\\ \Delta M_s &= \Delta M_s(SM)[1+f],\\ |\epsilon| &= \frac{G_F^2 f_K^2 M_K M_W^2}{6\sqrt{2}\pi^2 \Delta M_K}\hat{B}_K\left(A^2\lambda^6\bar{\eta}\right)\\ &\quad\times\big(y_c\left\{\hat{\eta}_{ct}f_3(y_c,y_t)-\hat{\eta}_{cc}\right\}\\ &\quad+\hat{\eta}_{tt}y_t f_2(y_t)[1+f]A^2\lambda^4(1-\bar{\rho})\big). \qquad (9)\end{aligned}$$

The parameter f is positive definite, so that the supersymmetric contributions add *constructively* to the SM contributions in the entire allowed supersymmetric parameter space. The size of f depends, in general, on the parameters of the supersymmetric model. In our fits, we will consider four representative values of f – 0, 0.2, 0.4 and 0.75 – which are typical of the SM, minimal supergravity (SUGRA) models, non-minimal SUGRA models, and non-SUGRA models, respectively.

For the SUSY fits, we use the same program as for the SM fits, except that the theoretical expressions for ΔM_d, ΔM_s and $|\epsilon|$ are modified as above (9). The allowed 95% C.L. regions for the four values $f = 0$, 0.2, 0.4, and 0.75 are all plotted in Fig. 4. We can see from this figure that, as f increases, the allowed region moves slightly down and towards the right in the $\bar{\rho}$–$\bar{\eta}$ plane.

At present, there is still considerable overlap between the $f = 0$ (SM) and $f = 0.75$ regions. However, there are also regions allowed for one value of f which are excluded for another value. In particular, one notices that, as f increases, larger values of $\bar{\rho}$ are allowed. This in turn implies that larger values of ΔM_s are allowed, and in fact this is borne out quantitively. The allowed ranges for ΔM_s (95% C.L.) are given by:

$$\begin{aligned} f = 0\ (\mathrm{SM}) &: 14.6 \le \Delta M_s \le 31.2\ ,\\ f = 0.2 &: 14.6 \le \Delta M_s \le 35.5\ ,\\ f = 0.4 &: 14.9 \le \Delta M_s \le 39.4\ ,\\ f = 0.75 &: 15.1 \le \Delta M_s \le 48.6\ . \qquad (10)\end{aligned}$$

Although the lower limit on ΔM_s is roughly independent of f, the upper limit increases as f increases. Thus, should ΔM_s be found to be very large, this would be consistent with SUSY models with large values of f. Conversely, if ΔM_s is measured to be near its lower limit, this would disfavor SUSY models with large f. (Note that, although small values of ΔM_s are allowed in such models, the region of parameter space which yields such values is relatively small. Thus, one can expect the fits to the data to be poorer for SUSY models with large values of f than for models with small f. We will see this in more detail in Sect. 3.)

As was seen in the SM fit, different shapes of the unitarity triangle correspond to different values of the CP phases α, β and γ. Furthermore, these allowed values are correlated: the correlations between $\sin 2\alpha$ and $\sin 2\beta$, and between α and γ, are shown in Fig. 2 and Fig. 3, respectively. Tables 2 and 3 give, respectively, the allowed ranges for the CP phases and the quantities measured in CP-violating asymmetries. The key observation here is that a

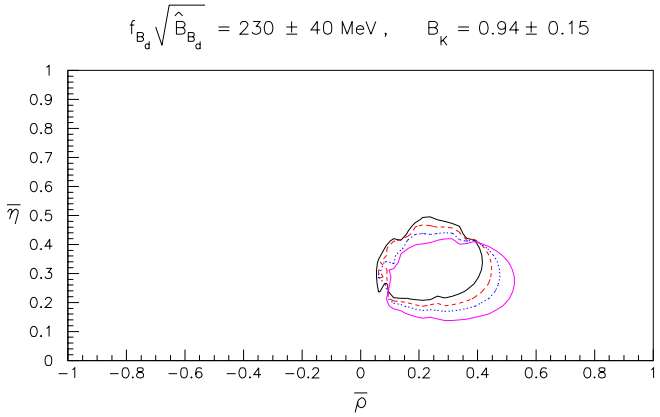

Fig. 4. Allowed 95% C.L. region in ρ–η space in the SM and in SUSY models, from a fit to the data given in Table 1. From left to right, the allowed regions correspond to $f = 0$ (SM, solid line), $f = 0.2$ (long dashed line), $f = 0.4$ (short dashed line), $f = 0.75$ (dotted line)

Table 2. Allowed 95% C.L. ranges for the CP phases α, β and γ, as well as their central values, from the CKM fits in the SM ($f = 0$) and supersymmetric theories, characterized by the parameter f defined in the text

f	α	β	γ	$(\alpha,\beta,\gamma)_{\rm cent}$
$f = 0$ (SM)	77° – 127°	14° – 35°	34° – 81°	(95°, 22°, 63°)
$f = 0.2$	80° – 133°	13° – 34°	29° – 81°	(109°, 22°, 49°)
$f = 0.4$	82° – 138°	12° – 34°	25° – 78°	(112°, 20°, 48°)
$f = 0.75$	87° – 146°	10° – 35°	20° – 74°	(114°, 21°, 45°)

Table 3. Allowed 95% C.L. ranges for the CP asymmetries $\sin 2\alpha$, $\sin 2\beta$ and $\sin^2\gamma$, from the CKM fits in the SM ($f = 0$) and supersymmetric theories, characterized by the parameter f defined in the text

f	$\sin 2\alpha$	$\sin 2\beta$	$\sin^2\gamma$
$f = 0$ (SM)	−0.96 – 0.45	0.46 – 0.94	0.31 – 0.98
$f = 0.2$	−1.00 – 0.35	0.44 – 0.93	0.24 – 0.97
$f = 0.4$	−1.00 – 0.26	0.42 – 0.93	0.19 – 0.96
$f = 0.75$	−1.00 – 0.11	0.36 – 0.94	0.12 – 0.93

measurement of the CP angle β will *not* distinguish among the various values of f – the allowed range for β is rather independent of f. If one wants to distinguish among the various SUSY models, it will be necessary to measure α and/or γ. (Of course, as mentioned above, there is still significant overlap among all four models. Thus, depending on what values of α and γ are obtained, we may or may not be able to rule out certain values of f.)

3 Unitarity triangle: Future profile

As was discussed in Sect. 2.1, the B_s^0–$\overline{B_s^0}$ mixing data appears to contain a 2.5σ signal centered at $\Delta M_s = 17.7\ \mathrm{ps}^{-1}$. This signal is not statistically significant enough

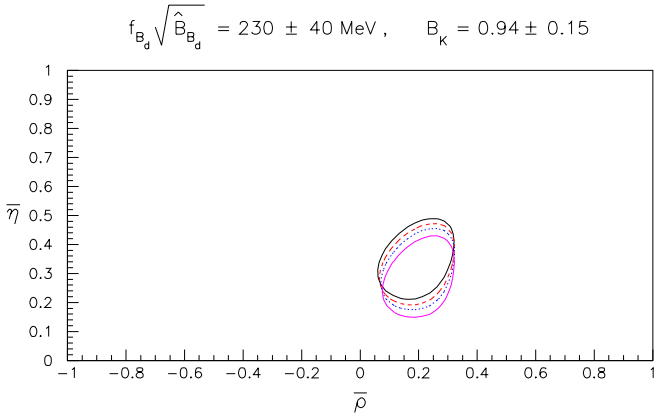

Fig. 5. Allowed 95% C.L. region in ρ–η space in the SM and in SUSY models, in the hypothetical scenario in which ΔM_s is given by (11). From top to bottom, the allowed regions correspond to $f = 0$ (SM, solid line), $f = 0.2$ (long dashed line), $f = 0.4$ (short dashed line), $f = 0.75$ (dotted line)

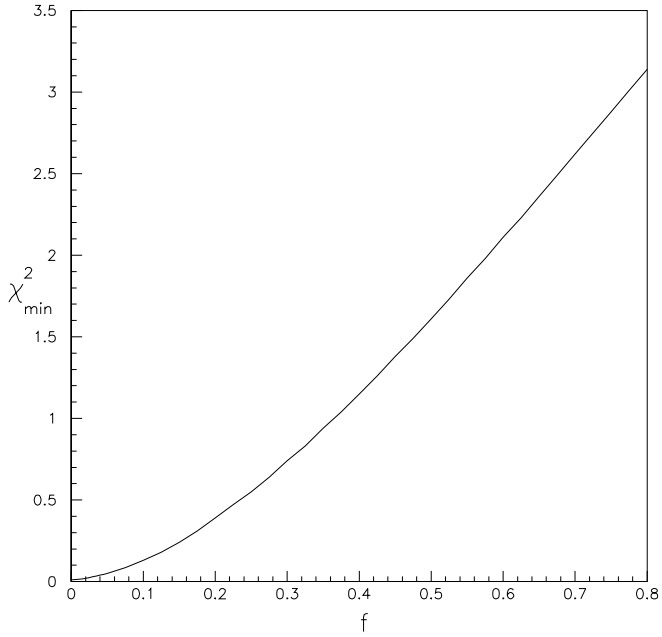

Fig. 6. Minimum value of χ^2 as a function of the SUSY parameter f, for the fits in the hypothetical scenario in which ΔM_s is given by (11). Models with $\chi^2_{min} > 2$ are disfavored

to be considered a measurement of ΔM_s. However, it is still interesting to consider what the effect would be on the profile of the unitarity triangle, both in the SM and in SUSY models, if this signal persisted and became a measurement. This is the purpose of this section.

In order to be consistent with both the central value of ΔM_s and its 95% C.L. lower limit (14.9 ps^{-1}), we assume the hypothetical future measurement of this quantity to be

$$\Delta M_s = 17.7 \pm 1.4 \text{ ps}^{-1} \ . \tag{11}$$

The SM and SUSY fits are then performed with this as part of the input data.

The results are shown in Fig. 5. The effect of the ΔM_s constraint is quite striking: the minimum- and maximum-allowed values of $\bar{\rho}$ are essentially independent of f. Now, as f increases, the allowed region only moves slightly down in the $\bar{\rho}$–$\bar{\eta}$ plane.

However, Fig. 5 does not tell the whole story. In particular, it does not take into account how good the fits are for the various values of f. The goodness of fit is indicated by the minimum value of χ^2: since there are two degrees of freedom ($\bar{\rho}$ and $\bar{\eta}$), fits with $\chi^2_{min} > 2$ are disfavored. In fact, the model with $f = 0.75$ has $\chi^2_{min} = 2.9$, and is hence a poor fit to the data. In Fig. 6 we present χ^2_{min} as a function of f. This figure shows that, for the hypothetical scenario in which ΔM_s is given by (11), models with $f > 0.6$ are disfavored.

It is interesting – and perhaps somewhat discouraging – to note that the best fit ($\chi^2_{min} = 9.5 \times 10^{-3}$) occurs for $f = 0$, i.e. for the standard model. That is, although some models with $f \neq 0$ would give reasonable fits to the data, the hint of a signal at $\Delta M_s = 17.7$ ps^{-1} does not indicate any problems whatsoever for the SM.

Note also that the percentage error we have assumed for ΔM_s, 7.9%, is considerably greater than the present experimental error on ΔM_d of 2.9%. It is not unreasonable to believe that the percentage error on ΔM_s will eventually approach that of ΔM_d. In that case, the precise measurement of ΔM_s will be able to rule out an even greater region of SUSY parameter space. That is, values of f smaller than 0.6 will be disfavored. Thus, we see that a precision measurement of ΔM_s will be an extremely powerful tool for distinguishing among the SM and its various supersymmetric extensions.

Table 4. Allowed 95% C.L. ranges for the CP phases α, β and γ, as well as their central values, from the CKM fits in the SM ($f = 0$) and supersymmetric theories, in the hypothetical scenario in which ΔM_s is given by (11)

f	α	β	γ	$(\alpha, \beta, \gamma)_{\text{cent}}$
$f = 0$ (SM)	80° – 119°	14° – 34°	44° – 79°	(98°, 22°, 60°)
$f = 0.2$	82° – 123°	13° – 33°	41° – 77°	(101°, 21°, 58°)
$f = 0.4$	84° – 127°	12° – 32°	38° – 76°	(105°, 20°, 55°)
$f = 0.75$	87° – 134°	10° – 30°	34° – 73°	(110°, 18°, 52°)

For completeness, in Figs. 7 and 8 we present, respectively, the $\sin 2\alpha$–$\sin 2\beta$ and α–γ correlations for the scenario in which ΔM_s is given by (11). The allowed ranges for the CP phases and for $\sin 2\alpha$, $\sin 2\beta$ and $\sin^2\gamma$ are given in Tables 4 and 5, respectively. A comparison of, for example, Tables 2 and 4 reveals that, as expected, the measurement of ΔM_s does not affect the allowed range for β appreciably, though the ranges for α and γ are significantly reduced.

4 Conclusions

The latest experimental data on B^0_s–$\overline{B^0_s}$ mixing puts the 95% C.L. lower limit at $\Delta M_s > 14.9$ ps^{-1}. Furthermore, there is an intriguing 2.5σ hint of a signal at $\Delta M_s \simeq$

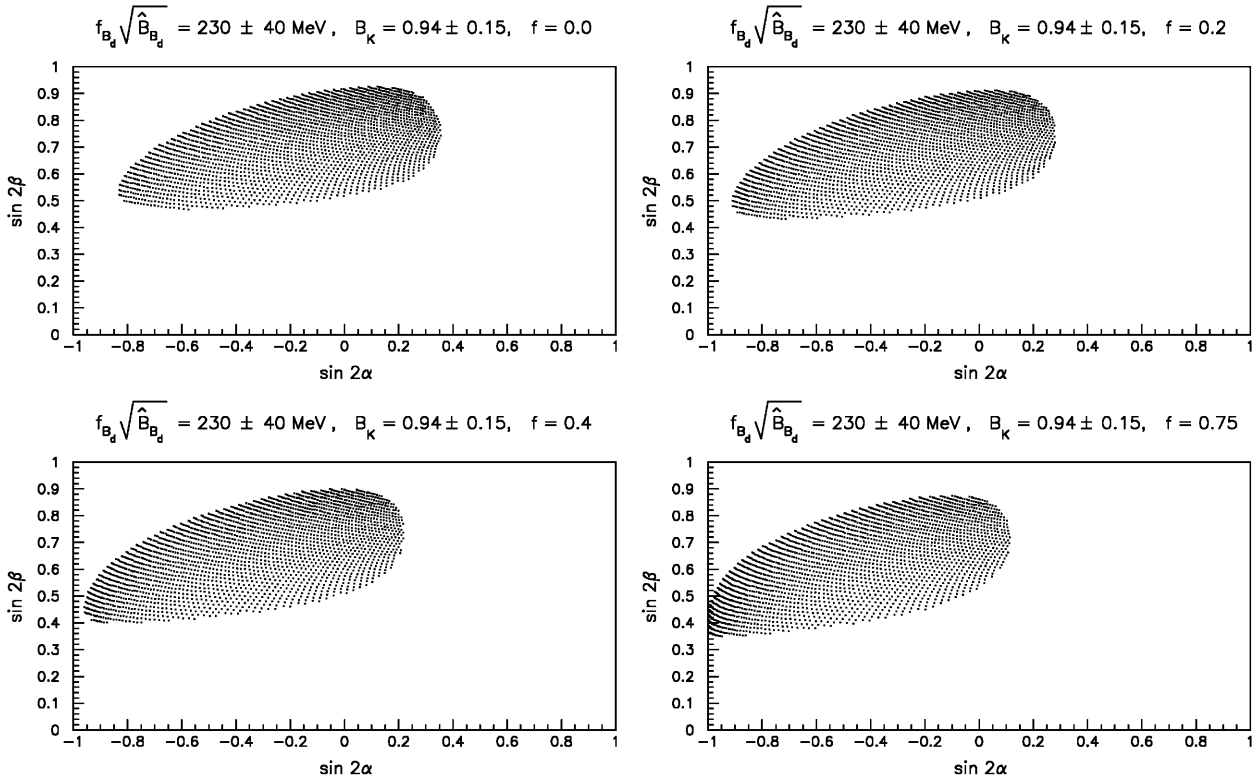

Fig. 7. Allowed 95% C.L. region of the CP-violating quantities $\sin 2\alpha$ and $\sin 2\beta$, in the hypothetical scenario in which ΔM_s is given by (11). The upper left plot ($f = 0$) corresponds to the SM, while the other plots ($f = 0.2$, 0.4, 0.75) correspond to various SUSY models

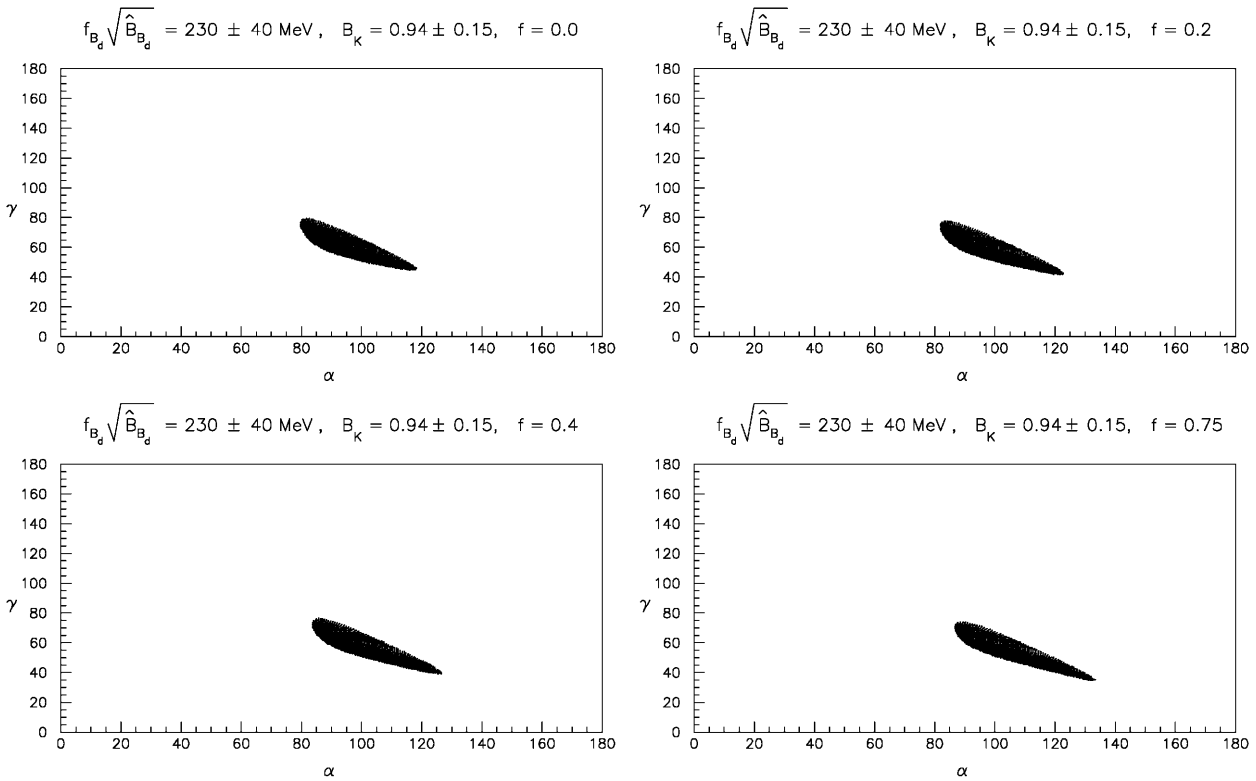

Fig. 8. Allowed 95% C.L. region of the CP-violating quantities α and γ, in the hypothetical scenario in which ΔM_s is given by (11). The upper left plot ($f = 0$) corresponds to the SM, while the other plots ($f = 0.2$, 0.4, 0.75) correspond to various SUSY models

Table 5. Allowed 95% C.L. ranges for the CP asymmetries $\sin 2\alpha$, $\sin 2\beta$ and $\sin^2\gamma$, from the CKM fits in the SM ($f = 0$) and supersymmetric theories, in the hypothetical scenario in which ΔM_s is given by (11)

f	$\sin 2\alpha$	$\sin 2\beta$	$\sin^2\gamma$
$f = 0$ (SM)	-0.84 – 0.35	0.47 – 0.93	0.48 – 0.96
$f = 0.2$	-0.92 – 0.27	0.44 – 0.91	0.43 – 0.95
$f = 0.4$	-0.97 – 0.20	0.40 – 0.90	0.38 – 0.94
$f = 0.75$	-1.00 – 0.09	0.35 – 0.87	0.31 – 0.92

17.7 ps^{-1}. In light of this, in this paper we examine the effect that a measurement of ΔM_s would have on the profile of the CKM matrix, both in the standard model and in supersymmetric models with minimal flavor violation.

We first update the profile of the unitarity triangle, both in the SM and in supersymmetric models, using current experimental data. The SUSY contributions to ΔM_d, ΔM_s and $|\epsilon|$ can all be described by a single common parameter f, and we take three representative values in our fits: $f = 0.2$, 0.4 and 0.75. The measurement of the CP-phase β will not distinguish among the various models, though the measurement of α and/or γ may do so. More importantly, the different models make different predictions for the allowed range of ΔM_s. This indicates that the measurement of ΔM_s will also be important for distinguishing among the various models.

This point is made quantitatively when the fits are repeated assuming a hypothetical measurement of $\Delta M_s = 17.7 \pm 1.4$ ps^{-1}. In this case, we find that SUSY models with $f > 0.6$ provide poor fits to the data, and are hence disfavored. Thus, we see that the measurement of ΔM_s is indeed a powerful tool for discriminating between the SM and its supersymmetric extensions.

For the particular experimental value of ΔM_s that we have assumed – and we have chosen this value to be consistent with the lower 95% C.L. bound, as well as with the hint of a signal – the best fit to the data occurs for $f = 0$, i.e. for the SM. Thus, present data on B_s^0–$\overline{B_s^0}$ mixing does not indicate any problems with the SM, which may be somewhat discouraging for those who hope to see signals for new physics via CKM phenomenology.

Finally, it is not unreasonable to expect that the percentage error on a measurement of ΔM_s will eventually reach the same level as that of ΔM_d (i.e. $\sim 3\%$). When this happens, the precise measurement of ΔM_s will be able to rule out an even greater region of SUSY parameter space. (Or, if the central value changes, one could conceivably rule out the SM!) Once again, this emphasizes the importance of a measurement of ΔM_s for searching for new flavor physics.

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